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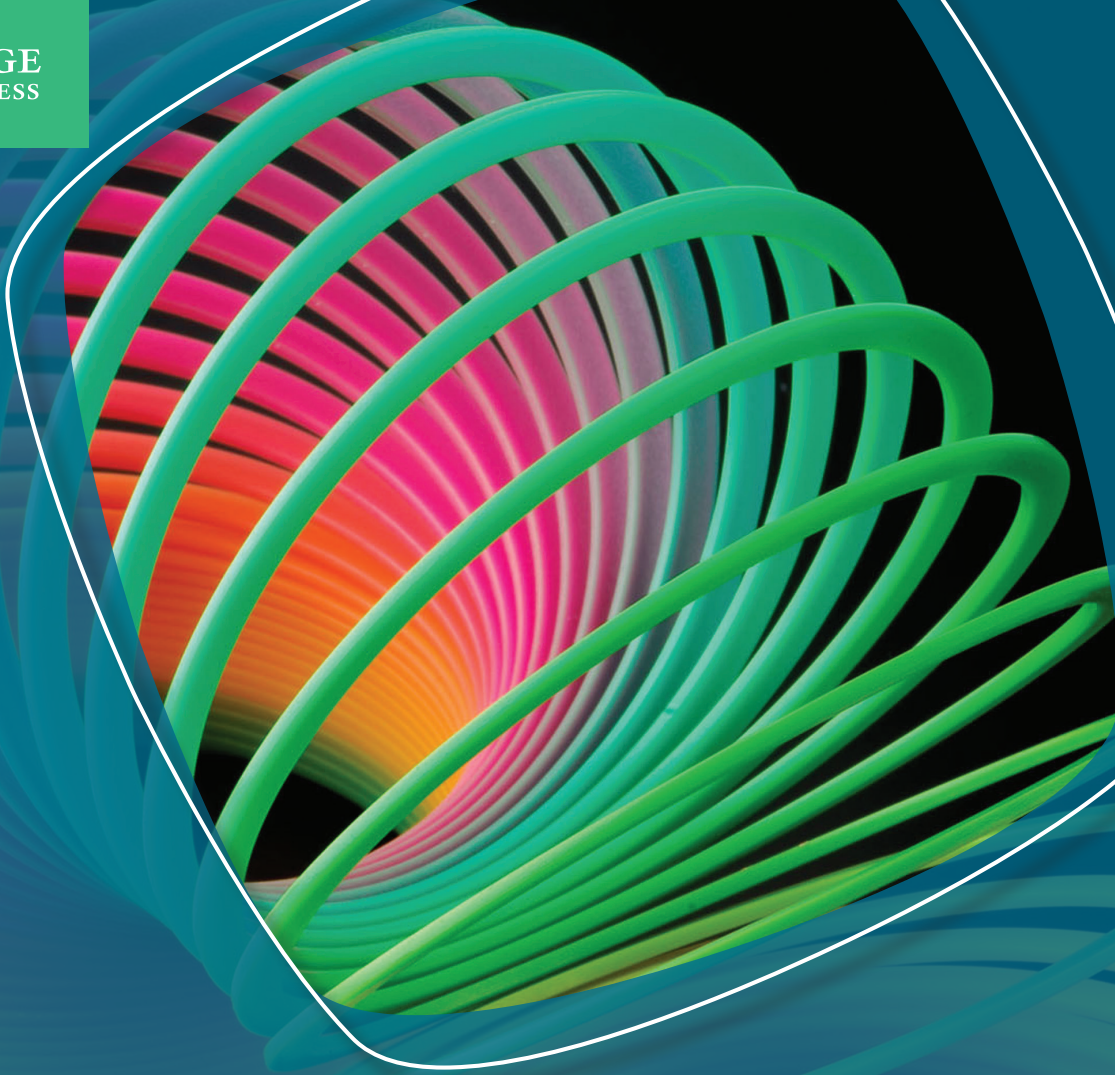

Mathematical Studies Standard Level

for the IB Diploma

Exam Preparation Guide

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INTRODUCTION

ABOUT THIS BOOK

If you are using this book, you're probably getting quite close to your exams. You may have started off as a bright-eyed student keen to explore international perspectives in mathematics and the nature of mathematical knowledge, but now you want to know how to get the best possible grade! This book is designed to revise all of the material that you need to know, and to provide examples of the most common types of exam questions for you to practise, along with some hints and tips regarding exam technique and common pitfalls.

Any common pitfalls and useful exam hints will be highlighted in these boxes.

This type of box will be used to point out where graphical calculators can be used effectively to simplify a question or speed up your work. Common calculator pitfalls will also be highlighted in such boxes.

If the material in a chapter involves maths outside of that chapter, this kind of box will direct you to the relevant part of the book where you can go and remind yourself of the relevant maths.

The most important ideas and formulae are emphasised in the 'What you need to know' sections at the start of each chapter. When a formula or set of formulae is given in the Formula booklet, there will be a book icon next to it. If formulae are not accompanied by such an icon, they do **not** appear in the Formula booklet and you may need to learn them or at least know how to derive them.

For Mathematical Studies Standard Level, each of the written papers:

- is worth 40% of the final grade
- is one and a half hours long (plus 5 minutes of reading time)
- has a total of 120 marks available.

The difference between the two papers is that Paper 1 consists of 15 short questions while Paper 2 consists of 4 or 5 long questions which require you to use and connect several different ideas.

IMPORTANT EXAM TIPS

- **Grab as many marks as you can.**
    - If you cannot do an early part of a question, write down a sensible answer and use that in later parts or, if the part you could not do was a 'show that' task, use the given result. You will still pick up marks.

- Do not throw away 'easy marks':
  - Where appropriate, give units with your answers.
  - Give all answers exactly or to three significant figures, or to the accuracy specified in the question. Each time you fail to do so you will lose a mark.
  - Do not use rounded intermediate values, as this can result in an inaccurate answer; store all intermediate values in your calculator.
  - Read the questions carefully and make sure you have provided the answer requested. For example, if the question asks for coordinates, give both x and y values. If the question asks for an equation, make sure that you have something with an equals sign in it.
  - Do not leave anything blank; you won't be given negative marks for wrong answers, and you may pick up a mark for writing down something sensible.
  - Do not cross out your working unless you have already replaced it with something else.
- **The questions are actually worded to help you.**
  - Make sure you know what each command term means. (These are explained in the IB syllabus.) For example:
    - 'Write down' means that there does not need to be any working shown. So, for this type of question, if you are writing out lines and lines of algebra, you have missed something.
    - 'Hence' means that you have to use the previous part somehow. You will not get full marks for a correct answer unless you explicitly show how you have used the previous part.
    - 'Hence or otherwise' means that you can use any method you wish, but it's a pretty big hint that the previous part will help in some way.
    - 'Sketch' means that you do not need to do a precise and to-scale drawing; however, you should label all the important points and at the very least where the curve crosses any axes.
  - If the question refers to solutions, you should expect to get more than one answer.
  - Look out for links between the parts of a question, particularly in the long questions.
- **Use your 5 minutes of reading time effectively.**
  - Decide on the order in which you will attempt the questions. You do not have to answer them in order and you might want to do questions similar to ones you have seen before first. You will almost certainly want to do some parts of Section B before the last few questions in Section A.
  - Decide which questions can be done easily or checked effectively on the calculator. Do not be surprised if this is the majority of questions.
  - On Paper 2, think about how different parts of a question link together. There is often a difficult part in the middle of the question, but you can skip it and move on to the next part.


Practise using the reading time when attempting your practice papers.

Most importantly, there is nothing like good preparation to make you feel relaxed and confident going into an exam, which in turn should help you achieve your best possible result.

Good luck!

The author team

1 WORKING WITH NUMBERS

WHAT YOU NEED TO KNOW

- The definition of the different number sets:
  - $\mathbb{N}$ is the set of natural numbers $\{0, 1, 2, 3, \ldots\}$.
  - $\mathbb{Z}$ is the set of integers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$; $\mathbb{Z}^+$ is the set of positive integers; $\mathbb{Z}^-$ is the set of negative integers.
  - $\mathbb{Q}$ is the set of all rational numbers. A rational number can be written as a fraction using two integers, where the denominator must not be zero. Examples include $-\frac{1}{2}, 0.54, 17, 2\frac{3}{4}$.
  - $\mathbb{R}$ is the set of all real numbers. These include the natural numbers, integers and rational numbers, as well as irrational numbers. An irrational number cannot be written as a fraction; examples include π, $\sqrt{3}$, $\sin 60°$.
- Numbers can be written to a fixed number of decimal places or significant figures:
  - To round to one decimal place, look at the digit in the second decimal place; to round to two decimal places, look at the digit in the third decimal place. If that digit is less than 5, round down; if the digit is 5 or more, round up.
  - To round to three significant figures, look at the digit to the right of the third significant figure. If that digit is less than 5, round down; if the digit is 5 or more, round up.
- Very small and very large numbers can be expressed in standard form (or scientific notation) $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.
- To find the percentage error, use the formula $\varepsilon = \left|\frac{v_A - v_E}{v_E}\right| \times 100\%$, where v_E is the exact value and v_A is the approximate value of v.
- Converting to a larger unit means fewer of them, so divide. Converting to a smaller unit means more of them, so multiply.
- To change one currency to another, multiply by the appropriate exchange rate. When commission is charged, first work out the amount of commission paid. The exchange rate is then applied to the 'original amount − commission paid'.

EXAM TIPS AND COMMON ERRORS

- The square root of any number that is not a perfect square (e.g. $\sqrt{4}$) or the ratio of two perfect squares (e.g. $\sqrt{\frac{16}{25}}$) will be irrational. Many trigonometric ratios (e.g. $\sin 45°$) are irrational.
- Zeros at the beginning of a decimal (e.g. **0.000**301) and at the end of a large number (e.g. 134**000**) do not count as significant figures. The first significant figure of a number is the first non-zero digit in the number, counting from the left.

1.1 DIFFERENT TYPES OF NUMBERS

WORKED EXAMPLE 1.1

Write down a number that is:

(a) a real number but not rational

(b) a rational number and in $\mathbb{N}$

(c) a rational number but not in $\mathbb{N}$.

(a) π

π is an irrational number as it cannot be written as a fraction.

(b) 3

All integers are rational numbers, e.g. $3 = \frac{3}{1}$ is a fraction with denominator 1.

(c) $\frac{1}{2}$

Practice questions 1.1

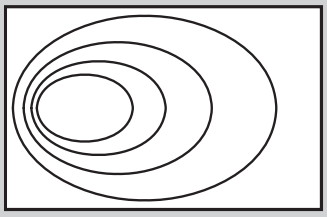
1. Mark each cell to indicate which number set(s) the number belongs to. The first row has been completed for you.

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{Z}$
-3	×	✓	✓	✓
0.76				
$\cos 120°$				
$\sqrt{5}$				
1.23×10^8				

2. The Venn diagram shows the sets $\mathbb{N}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{Z}$.

(a) Add labels to show which region corresponds to each set.

(b) Put the following numbers into the correct set:
3, -3, 3.3 and π.



3. A set contains the elements x such that $-4 \leq x < 6$, $x \in \mathbb{Z}$. Each element is equally likely to occur. One element is drawn at random.

Find the probability that it is in: (a) $\mathbb{N}$ (b) $\mathbb{Q}$.

1.2 STANDARD FORM AND SI UNITS

WORKED EXAMPLE 1.2

If $x = 2.3 \times 10^{12}$ cm and $y = 4.8 \times 10^{-11}$ km:

(a) write x and y in metres

(b) find xy, giving your answer in metres squared in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

1 km = 1000 m, 1 cm = 0.01 m, 1 mm = 0.001 m
1 kg = 1000 g, 1 mg = 0.001 g
$1\text{ m}^2 = 100\text{ cm} \times 100\text{ cm} = 10\,000\text{ cm}^2$
$1\text{ m}^3 = 100\text{ cm} \times 100\text{ cm} \times 100\text{ cm} = 1\,000\,000\text{ cm}^3$.

(a) $x = \dfrac{2.3 \times 10^{12}\text{ cm}}{100\text{ cm/m}} = 2.3 \times 10^{10}\text{ m}$

$y = 4.8 \times 10^{-11}\text{ km} \times 1000\text{ m/km} = 4.8 \times 10^{-8}\text{ m}$

(b) $xy = 2.3 \times 10^{10}\text{ m} \times 4.8 \times 10^{-8}\text{ m}$
$= 1104\text{ m}^2 \approx 1.10 \times 10^3\text{ m}^2$

Converting to a larger unit means fewer of them, so divide. Converting to a smaller unit means more of them, so multiply.

Make sure that you can convert numbers to standard form on your calculator. However, you must not write calculator notation, such as 10E3, in your answer.

Practice questions 1.2

4. Given that $a = 5.6 \times 10^{10}$ and $b = 1.6 \times 10^{-4}$, calculate the following, giving your answer in the form $c \times 10^k$ where $1 \leq c < 10$ and $k \in \mathbb{Z}$:

(a) ab (b) $\dfrac{a}{b}$

5. Given that $x = 4.3 \times 10^8$ g and $y = 0.98$ hours, calculate $\dfrac{x}{y}$, giving your answer in kg per second in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

6. A room measures 3.1 m by 4.4 m. Find the area of the room in:

(a) m^2 (b) cm^2

(c) Give your answer to (b) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

If numbers are very big or very small, the GDC gives the answers in standard form automatically.

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1.3 APPROXIMATION AND ESTIMATION

WORKED EXAMPLE 1.3

(a) Write down $\sqrt{2}$ correct to two decimal places.
(b) Write down $\sqrt{2}$ correct to the nearest ten.
(c) Write down $\sqrt{2}$ correct to two significant figures.
(d) Calculate the percentage error if $\sqrt{2}$ is given correct to two significant figures.

> In an exam question, if a specific degree of accuracy is not asked for, give your answer correct to three significant figures.

(a) 1.41

> $\sqrt{2} = 1.41421\ldots$. To round to 2 decimal places, look at the digit in the third decimal place, which is 4. It is less than 5, so round down.

(b) 0

> Remember that 0 is a multiple of ten.

(c) 1.4

> Find the second significant figure and look at the digit after it. As $1 < 5$, round down.

(d) $$\varepsilon = \left|\frac{v_A - v_E}{v_E}\right| \times 100\% = \left|\frac{1.4 - \sqrt{2}}{\sqrt{2}}\right| \times 100\%$$
$$= 1.01\% \text{ (3 SF)}$$

> Substitute the rounded value, $v_A = 1.4$, and the exact value, $v_E = \sqrt{2}$, into the formula for percentage error. The modulus sign simply means that we remove any negative sign which occurs.

Practice questions 1.3

7. (a) Write π correct to three decimal places.
(b) Find the percentage error when π is given correct to three decimal places.

8. (a) Write down $\sqrt{46}$ and π correct to three significant figures.
(b) Write down the value of $\pi^{\sqrt{46}}$, giving all the digits shown on your calculator.
(c) Write down the value of $\pi^{\sqrt{46}}$ using the approximate values found in part (a), giving all the digits shown on your calculator.
(d) To how many significant figures is your result in part (c) correct?
(e) What is the percentage error in your answer to part (c)?

1.4 CURRENCY CONVERSIONS

WORKED EXAMPLE 1.4

The table shows the exchange rates for US dollars and euros:

	? USD	? EUR
1 USD	1	0.78
1 EUR	p	1

(a) Find the value of p to two decimal places.

(b) What is the value of \$150 in euros?

(c) Jamie changes €300 into dollars. She is charged 6% commission. How many dollars does she receive?

(a) 1 USD = 0.78 EUR
$1 \div 0.78$ USD = 1 EUR
so $p = 1.28205\ldots = 1.28$ (2 DP)

We treat this as an equation and divide both sides by 0.78.

(b) \$150 = 150×0.78 EUR = 117 EUR

Use the exchange rate 1 USD = 0.78 EUR.

(c) $0.06 \times 300 = 18$ EUR
$300 - 18 = 282$ EUR
$282 \times 1.28205\ldots = 361.54$ USD

First work out the amount of commission paid. The exchange rate is then applied to the remaining amount.

Use the full accuracy of the conversion rate, not a rounded result.

Practice questions 1.4

9. The table gives the conversion rates between British pounds (GBP) and Australian dollars (AUD). Find x, and hence find the number of pounds which can be bought with 1000 AUD if there is a 4% commission.

	? GBP	? AUD
1 GBP	1	1.58
1 AUD	x	1

10. A currency office buys one South African rand (SAR) for 10.95 Japanese yen and sells one SAR for 11.05 yen.

(a) Blaise converts 1000 SAR to yen for a holiday. While on holiday he spends half of this money. On his return he converts the remainder back to SAR. How many SAR will he get back?

(b) Robbie also converts 1000 SAR to yen for a holiday. He then cancels his holiday and changes all the yen back to SAR. How much has Robbie lost after the two transactions? Express your answer as a percentage of Robbie's original 1000 SAR.

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Mixed practice 1

1. If $x = 1.23 \times 10^5$ and $y = 1.46 \times 10^{-3}$:

(a) write y to four decimal places

(b) calculate xy, giving your answer in decimal form

> ⚠ 'Decimal form' means as an ordinary number.

(c) calculate $\frac{y}{x}$, giving your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

2. Look at this list of numbers:

3.14×10^2, 100π, $\frac{2200}{7}$, $\sqrt{100\,000}$, 310

(a) Which of these numbers is largest?

(b) List all the numbers which are members of: (i) $\mathbb{Z}$ (ii) $\mathbb{Q}$.

(c) What is the largest number of significant figures to which all five numbers are equal?

3. A farmer wants to plant a new forest in a field covering a rectangular area of 3 km by 5 km.

(a) Find the area of the forest in m^2, giving your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(b) Each tree needs an area of $3.4\,m^2$. Find the maximum number of trees which could be planted. Give your answer to the nearest thousand trees.

4. A travel agent converts dollars and yen at an exchange rate of \$1 = 103 yen. They charge 5% commission on all transactions.

(a) If Dima converts 10 000 yen to dollars, how much will he receive?

(b) If he converts \$500 to yen, how much will he receive?

5. (a) Write 125.987 in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(b) Write 125.987 to two significant figures.

(c) What is the percentage error when rounding 125.987 to two significant figures?

6. Nicole wants to convert pounds to euros. She can choose between two different offers.

- Offer 1: an exchange rate of 1 pound to 1.26 euros with no commission.
- Offer 2: an exchange rate of 1 pound to 1.30 euros with 5% commission.

Which offer provides Nicole with the better deal?

7. Mark each cell to indicate which number set(s) the number belongs to. The first row has been completed for you.

Number	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
-5	×	✓	✓	✓
0				
$\tan 45°$				
$\tan 60°$				
9.9×10^{24}				
1×10^{-2}				
π				

8. The Earth can be modelled by a perfect sphere with a radius of 6700 km.

(a) Find the volume of the Earth in km^3.

(b) Find the volume of the Earth in cm^3, giving your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(c) If the average density of the earth is $6.7\ g/cm^3$, find the mass of the Earth.

> The formula for the volume of a sphere is given in the Formula booklet.

Going for the top 1

1. The value of x is quoted as 20 to the nearest 10. The value of y is quoted as 1.6 to two significant figures.

(a) Write an inequality in the form $a \leq x < b$ showing the range that x can lie in.

(b) What is the smallest value $3x - 7$ could take?

(c) Find the largest possible percentage error if $\frac{x}{y}$ is quoted as being 12.5.

2. The table below shows the values of different currencies compared to one US dollar:

	GBP	CHF	EUR	JPY	AUD
1 USD	0.66	0.97	0.78	102.48	1.02

(a) How many US dollars can you get with one British pound (GBP)?

(b) What is the exchange rate for euros (EUR) to Japanese yen (JPY)?

(c) Camille converts 1000 US dollars to British pounds, then Swiss francs (CHF), then euros, then Japanese yen, then Australian dollars, and then back to US dollars. For each transaction, she pays 5% commission. How much does she have left at the end, to the nearest US dollar?